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 Poynting Vector

Suppose there is a region of space where an electric field \vec{E} cause a current flow of density \vec{J} . Then the power dissipated per unit volume of $E \cdot J$ and the total power dissipated will be

$$\int \vec{E} \cdot \vec{J} dt = \int \vec{E} \cdot \left(\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} \right) dt$$

$$\text{As } \nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

Therefore power dissipated is

$$\begin{aligned} &= S \left[\vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} + \nabla \cdot (\vec{E} \times \vec{H}) \right] dt \\ &= \frac{1}{2} \frac{\partial}{\partial t} \int (\vec{D} \cdot \vec{E} + \vec{B} \cdot \vec{H}) dt - \int \nabla \cdot (\vec{E} \times \vec{H}) dt \end{aligned}$$

Transforming the volume integral into surface integral over the surface enclosing the volume. The first term represents the rate at which the energy stored in the electromagnetic field diminishes and which the energy flows ~~outward~~ into the ~~unbounded~~ volume under consideration.

Thus $\frac{1}{2}(\bar{D} \cdot \bar{E} + \bar{B} \cdot \bar{H})$ is the energy density of the electromagnetic field and the vector $\bar{P} = \bar{E} \times \bar{H}$ is the rate at which energy flows across unit area of the boundary when an electromagnetic wave is propagated.

$\bar{P} \times \bar{E} \times \bar{H}$ is known as Poynting Vector.
